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AN INTERFACE RELAXATION FEM-BEM COUPLING METHOD WITH ELASTO-PLASTIC DEFORMATIONS IN THE FEM SUB-DOMAIN

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This paper presents an interface relaxation finite element-boundary element coupling method for elasto-plastic analysis. The domain of the original problem is decomposed into finite element and boundary element sub-domains where the elasto-plastic response is captured by the finite element sub-domain. Consistency of the sub-domains problem with the original one is ensured. The coupling method does not require access to the system matrices generated by the finite elements and boundary element models and, furthermore, is capable of handling natural boundary conditions applied at the entire real boundary of the finite element or boundary element sub-domains. In the example applications, the calculation results obtained by means of the presented method are compared with those by conventional finite element models.

Keywords: Boundary Element Method; Finite Element Method; Coupling; Interface Relaxation; Elasto-Plasticity.

1. Introduction

The finite element method (FEM) and the boundary element method (BEM) are powerful computational techniques for obtaining approximate solutions to the partial differential equations that arise in scientific and engineering applications. Each method has its own range of applications where it is most efficient and neither enjoys the distinction of being “the best” for all applications. The FEM is usually a method of choice in dealing with problems involving non-linearity in domains of finite dimensions while the BEM is efficient, accurate and relatively easy to use in treatment of linear semi-infinite and infinite domains. Thus, if the problem of interest includes local non-linearity only in a portion of the infinite domain, the concept of solving it in an adjacent sub-domains, employing the most suited solution technique for each of them, is, by all means, appropriate.

The general technique of FEM-BEM coupling was developed in a classic paper by Zienkiewicz et al. [1]. The theory and algorithms of coupling FEM and BEM solutions reached, by now, a fairly matured state. The conventional FEM-BEM coupling methods employ a unified, global set of equations for the entire domain by combining the discretized equations from the BEM and FEM sub-domains. The conventional coupling methods, however, may destroy the desirable features originally existing in the FEM matrices, namely, symmetry, sparsity and bandedness. Moreover, implementation itself needs an integrated *finite element-boundary element* computational environment and results in the highly undesirable requirement of accessing their source codes.

Recently, the domain decomposition algorithms have been utilized to couple the FEM and BEM methods [2-4]. Unlike the conventional approaches, the domain decomposition FEM-BEM coupling methods have physical interpretation and are relatively easy to comprehend. Final solution is obtained iteratively as FEM and BEM sub-models are solved separately with successive updates of degrees of freedom at the interface of sub-domains, until convergence there is reached. Existing domain decomposition coupling methods [2-4] set the natural boundary condition on interface (approached from the FEM and/or BEM sub-domains). Unfortunately, in situations when at the remaining part of sub-domain boundary only natural BC can be imposed, this plainly leads to singularity of matrix/matrices and non-unique solution. An example of such problem is a local non-linearity (to be dealt with FEM) in an infinite domain (to be dealt with BEM). This problem can be partially alleviated employing the overlapping domain decomposition methods [5, 6]. The overlapping, on the other hand, may create serious complication in the Schwarz method, even when the global problem is that of a simple geometry.

Far more general than the standard domain decomposition are the interface relaxation methods [7, 8]. They have all of the advantages of the domain decomposition methods and, furthermore, allow one to handle unrelated PDE problems within different sub-domains. In references [9, 10] Elleithy and Tanaka presented several interface relaxation algorithms for coupling the FEM and BEM. The interface relaxation coupling method presented in [10] estimates new values of the