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RECONSTRUCTION OF MULTIDIMENSIONAL BOUNDARY CONDITION DISTRIBUTION IN ELASTICITY PROBLEMS USING FEM-BASED FILTERING TECHNIQUE

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In this paper we discuss the FEM-based filtering technique used for the reconstruction of multidimensional boundary condition (BC) distribution in elasticity and thermoelasticity problems. The set of constraints equations is expressed in such a way it allows one to exploit the sub-matrices for sensitivity problem analysis, solved as a predictor step. Correction is achieved by employing filtering technique, which, moreover, ensures adherence of all estimated parameters to system global equilibrium. Strain components and temperatures are used as input data subject to uncertainties. In presented numerical examples the method is capable of reconstructing transient, distributed mechanical and thermal loads with reasonable accuracy.

Keywords: Inverse problems, sensitivity coefficients, filtering techniques

1. Introduction

In a number of practical applications some boundary conditions (BC) are unknown, as it is often impossible to place measurement devices, usually due to geometrical inaccessibility or exposure to severe environment conditions, at some portions of the domain of interest. Thus, the BC set is incomplete and to obtain solution an additional information about the considered process must be provided. That data in elasticity/thermoelasticity problems is usually obtained via measured strain components (using strain gauges) and/or temperatures (using thermocouples). As all measured quantities are subject to unavoidable measurement errors/uncertainties and the inverse problems are always ill-posed, some regularization procedures must be implemented to prevent amplification of the errors contained in input data during the solution procedure. There is a wide range of procedures which can be implemented.

In this study we discuss an application of FEM-based filtering technique, in which regularization is inherent via covariance matrices a priori, to the estimation of multidimensional boundary conditions (BC). The constraint equations are formulated in such a way that entries to all required coefficient matrices may be obtained by means of commercial codes, as indicated in [1]. Furthermore, as some of sub-matrices of the formulation are equal to the sensitivity coefficient matrices, a combination of filtering and ordinary least squares (OLS)/orthogonal distance regression (ODR) formulations is available.

In this paper the weighted OLS solution is used as a predictor step, giving preliminary values of estimated quantities and their uncertainties a priori, while filter algorithm refines solution and ensures adherence of all estimated parameters to system global equilibrium. Moreover, it allows one, as in any such approach, to obtain a statistical measure of the accuracy of estimated field variables and BC(s).

The accuracy and stability of the algorithms is verified by considering the problem of inverse elasticity and thermoelasticity with simulated input data specified in the form of strain components and temperatures subject to random errors.

2. Problem formulation

Elastic deformation of an arbitrary solid is governed by differential equation, which in most of the practical applications is solved by means of numerical methods. If the *Finite Element Method* (FEM) is employed semi-discretization of it results in:

$$[\mathbf{S}]\{\boldsymbol{\delta}\}+[\boldsymbol{\Psi}]\{\dot{\boldsymbol{\delta}}\}+[\mathbf{M}]\{\ddot{\boldsymbol{\delta}}\}=\{\mathbf{L}\} \tag{1}$$

where $[\mathbf{S}]$, $[\boldsymbol{\Psi}]$, $[\mathbf{M}]$, $\{\mathbf{L}\}$, and $\{\boldsymbol{\delta}\}$ stand for stiffness, damping, mass matrices, load and displacements vectors, respectively (throughout the paper we will keep notation in which [ ] denotes rectangular/square matrix and { } vector). In many problems the structural dynamic effects are not of concern, inertial and damping structural effects can be ignored, and the set of equations (1) simplifies to:

$$[\mathbf{S}]\{\boldsymbol{\delta}\}=\{\mathbf{L}\} \tag{2}$$